The Norwegian University of Science and Technology
Department of Physics

Contact person:
Name: Jan Myrheim
Telephone: 93653

Examination, course FY2450 and MNFFY250 Astrophysics
Friday June 4, 2004
Time: 09.00–15.00

Grades made public: Friday June 25, 2004

Allowed to use: Calculator, mathematical tables.

A table of physical constants is found at the end of this set of problems.
The alphabetically numbered problems will be weighted equally.

Problem 1:

a) Sketch briefly, by means of an Hertzsprung–Russell diagram (HR diagram), how the Sun develops from it was formed from a gas cloud until it ends as a white dwarf.

b) When, where and how were the various chemical elements heavier than hydrogen formed? Give a brief answer.
Why are these processes only possible at high temperature (a few million K)?
Why are some elements, such as carbon and oxygen, much more abundant than others, such as gold and silver? In what way is iron special in this connection?

c) In our own galaxy, and in many others, we see clear differences between the oldest stars (population II) and the younger ones (population I).
Mention at least two important differences.
To which population does the Sun belong?

d) What distinguishes the open star clusters from the globular clusters?
Harlow Shapley used the globular clusters in order to localize the centre of our Milky Way galaxy, and to measure its distance How?

e) Shapley observed in particular a special type of variable stars, named after a star called RR Lyrae (because it was the tenth variable star observed in the constellation Lyra, the first nine being designated by the letters R to Z). The RR Lyrae stars may be used for measuring distances in the same way as a different type of variable stars called Cepheids, after the star δ Cephei.
On which property of these stars (both RR Lyrae stars and Cepheids) is the method based?

f) Sketch briefly the "distance ladder", that is, the different methods available for measuring distances. When calibrated against each other, these methods enable astronomers to measure distances in the universe, both to the nearest stars and to the most distant galaxies.

g) If you observe a pulsar, how can you know that it is a pulsar?
Mention a few good reasons that most astronomers believe that pulsars are neutron stars.

h) The magnetic flux density B on the surface of a neutron star may be 10^8 tesla, or even 10^{10} tesla in the case of a so called magnetar.
An electron in a magnetic field moves in a circle with an angular frequency $\omega = eB/m$ (called the synchrotron frequency), where e is the elementary charge, and m the electron mass. In quantum mechanics this means that the electron has a quantized energy which is an integer multiple of $\hbar\omega$.
Compute the so called Landau splitting $\hbar\omega$ for an electron in a field of 10^{10} T, and compare with the rest energy mc^2.

i) On Tuesday June 8, in four days, there will occur a Venus passage, then we may see Venus pass in front of the Sun during 6 hours (if the weather is clear). The previous Venus passage took place in 1882.
How many hours may a Venus passage last, at most?
The mean distance between the Sun and Venus is 0.723 astronomical units. The orbital period is 224.70 days for Venus and 365.256 days for the Earth.

j) The next Venus passage will take place already on June 6, 2012, that is, in 8 years.
Why 8 years?

k) A curious phenomenon is that Venus rotates about its own axis in such a way that it turns the same face to the Earth every time the two planets pass each other at a minimum distance. Unless this is a pure coincidence, it has to be caused by the tidal forces acting on Venus from the Earth. The tidal force from a planet of mass M at a distance r is proportional to GM/r^3 (because it is proportional to the gradient of the gravitational field GM/r^2). In practice this means that the tidal force on Venus from the Earth is negligible when the distance is largest, compared to when the distance is smallest.
Compare the tidal force on Venus from the Earth and from the Sun.
The tidal force from the Sun is much larger than the tidal force from the Earth, but what matters in this case is rather the variation of the tidal force from the Sun due to the variation of the distance to Venus. This variation might have caused Venus to have bound rotation relative to the Sun rather than relative to the Earth.
Compare the tidal force on Venus from the Earth with the variation in the tidal force from the Sun. The eccentricity of the orbit of Venus is $e = 0.0068$, meaning that the distance varies between $(1-e)a$ and $(1+e)a$, where a is the mean distance (the semimajor axis of the elliptical orbit).
Comment on the result of the comparison.

Problem 2:

The virial theorem holds for a physical system (such as a star, a planetary system, a gas cloud or a galaxy cluster) held together by gravitation, and being in a stationary state, in the sense that it neither expands nor contracts over a long time interval. The theorem says that

$$2E_K + V = 0 \,,$$

where E_K is the total kinetic energy, and V the total gravitational potential energy. The total mechanical energy of the system is

$$E = E_K + V \,.$$

These two equations together mean that $E_K = -E$ and $V = 2E$.

a) We may conclude from this that $E < 0$. Why?

We imagine a gas cloud consisting of N particles, with a total mass $M = N\overline{m}$, where $\overline{m}$ is the average mass per particle. The total kinetic energy of the cloud is

$$E_K = \frac{3}{2} N k_B T \,,$$

where k_B is Boltzmann's constant, and T is the average temperature.
We may assume that the gas cloud is spherically shaped with a radius R. Then the potential energy is

$$V = -a\,\frac{GM^2}{R} \,,$$

where G is Newton's gravitational constant, and a is a numerical factor close to 1, for example, $a = 3/5$ if the gas cloud has a constant mass density.

b) Over a short time interval (a few years) we may neglect energy loss due to radiation, and assume that the total energy $E = E_K + V$ is constant.
As said above, we must have $2E_K + V = 0$ in a stationary state. Will such a stationary state be stable as long as the total energy E is constant?
In order to investigate the stability, we may for example assume that $2E_K + V > 0$. Then this state can not be stationary, and since $E_K + E = 2E_K + V > 0$, the kinetic energy E_K is higher in this state than in a stationary state of given energy E. Hence we would expect the gas cloud to expand. What happens to the quantities E_K, V and $2E_K + V$ when the cloud expands with E constant?

c) Over a longer time interval (of a few hundred thousand years) the total energy E will be reduced, because energy is radiated away. At the same time as E is reduced, the gas cloud will all the time be in an approximately stationary state with $2E_K + V = 0$.
What happens then to the kinetic energy E_K, the potential energy V, the average temperature T, and the radius R?

d) It is believed that the galaxies were formed when the universe became transparent to electromagnetic radiation, about 300 000 years after the beginning, when the temperature had become low enough that neutral atoms could form. The temperature was then about 3000 K, and the density was 10^{-18} kg/m^3.
How much mass should a typical galaxy have, if the virial theorem held for a gas cloud consisting of 75 % neutral hydrogen and 25 % neutral helium (per centages of the mass), contracting to form a galaxy under these conditions? Compare with the solar mass.

e) Assume that the Sun was formed from such a spherical gas cloud, originally having a constant density and a temperature of 10 K. What radius and what mass density must the gas cloud have had originally, assuming that the virial theorem held all the time from the beginning of the collapse?

Some useful constants

Newton's gravitational constant:	$G = 6.673 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
The speed of light in vacuum:	$c = 299\,792\,458\,\mathrm{m/s}$
The permeability of vacuum:	$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2}$
The permittivity of vacuum:	$\epsilon_0 = 1/(\mu_0 c^2) = 8.854 \times 10^{-12}\,\mathrm{F/m}$
The reduced Planck's constant:	$\hbar = h/(2\pi) = 1.055 \times 10^{-34}\,\mathrm{J\,s}$
The elementary charge:	$e = 1.602 \times 10^{-19}\,\mathrm{C}$
The fine structure constant:	$\alpha = e^2/(4\pi\epsilon_0\hbar c) = 1/137.036$
Boltzmann's constant:	$k_B = 1.38 \times 10^{-23}$ J/K
The electron mass:	$m_\mathrm{e} = 9.109 \times 10^{-31}\,\mathrm{kg} = 0,511\,\mathrm{MeV}/c^2$
The proton mass:	$m_\mathrm{p} = 1.6726 \times 10^{-27}\,\mathrm{kg} = 938,28\,\mathrm{MeV}/c^2$
The neutron mass:	$m_\mathrm{n} = 1.6749 \times 10^{-27}\,\mathrm{kg} = 939,57\,\mathrm{MeV}/c^2$
The mass of the Earth:	$M_\oplus = 6.0 \times 10^{24}\,\mathrm{kg}$
The radius of the Earth:	$R_\oplus = 6.4 \times 10^{3}\,\mathrm{km}$
The mass of the Sun:	$M_\odot = 2.0 \times 10^{30}\,\mathrm{kg}$
The radius of the Sun:	$R_\odot = 7.0 \times 10^{5}\,\mathrm{km}$
The astronomical unit (distance to the Sun):	$1\,\mathrm{AU} = 1.50 \times 10^{8}\,\mathrm{km}$