# FY3403 Particle physics
# Problemset 2 fall 2008

NTNU
Institutt for fysikk

**Problem 2. Neutrino oscillations**
An electron antineutrino created in a radioactive process like $\beta$-decay,

$$n \longrightarrow p + e + \bar{\nu}_e, \tag{1}$$

is not believed to be a genuine particle (in the sense that every particle with self-respect should have a well-defined mass). To simplify mathematics we consider only two generations. Then the states of $\bar{\nu}_e$ and $\bar{\nu}_\mu$ are linear, orthonormal superpositions of two antineutrino states with definite mass,

$$\begin{pmatrix} |\,\bar{\nu}_e\,\rangle \\ |\,\bar{\nu}_\mu\,\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\,\bar{\nu}_1\,\rangle \\ |\,\bar{\nu}_2\,\rangle \end{pmatrix}, \tag{2}$$

where the antineutrinos $\bar{\nu}_1$ and $\bar{\nu}_2$ have (small) masses $m_1$ and $m_2$.

**a)** Assume now that $\bar{\nu}_e$ is created with a fixed momentum $\boldsymbol{p}$ (such that $|\boldsymbol{p}| \gg m_1 c,\, m_2 c$) at time $t = 0$, and denote this state by $|\,\Psi_{\boldsymbol{p}}(0)\rangle = |\,\bar{\nu}_e\,\rangle$. Find $|\,\Psi_{\boldsymbol{p}}(t)\rangle$ expressed by $|\,\bar{\nu}_1\,\rangle$, $|\,\bar{\nu}_2\,\rangle$ and the parameters $\theta$, $m_1$ and $m_2$.

**b)** After time $t$ the neutrino has moved a distance $L = ct$ (it moves at very nearly the speed of light, since $|\boldsymbol{p}| \gg m_1 c,\, m_2 c$). Assume there is a neutrino detector at that distance from the creation point, based on a process of inverse $\beta$-decay,

$$\bar{\nu}_e + X \longrightarrow e^+ + Y \tag{3}$$

where the nucleus $Y$ is related to the nucleus $X$ by the transformation of one proton into a neutron. To evaluate the probability for this one must express the state $|\,\Psi_{\boldsymbol{p}}(t)\rangle$ by the states $|\,\bar{\nu}_e\rangle$ and $|\,\bar{\nu}_\mu\rangle$. Do so to find the expression for the expansion coefficients $c_{\bar{e}}(t)$ and $c_{\bar{\mu}}(t)$. You may simplify the result by using the fact that

$$\sqrt{\boldsymbol{p}^2c^2 + m_i^2c^4} \approx |\boldsymbol{p}|c + \frac{m_i^2c^3}{2|\boldsymbol{p}|}.$$

**c)** Show that the probability of finding an electron antineutrino, $p_{\bar{e}}(t) = |c_{\bar{e}}(t)|^2$, is a simple oscillating function of $L$, and find an expression for the oscillation length $L_0$ (wavelength of oscillation).

**Problem 3. The particle content of the Standard Model**

**a)** List all the "building blocks" of matter particles in *the Standard Model for Elementary Particles*.

**b)** How are (i) *baryons*, (ii) *mesons*, (iii) *hadrons*, and (iv) *leptons* made out of these building blocks (to first approximation)?

**c)** According to the classification above, of which particle types are the following elementary particles: (i) $\pi^+$, (ii) $\Delta^-$, (iii) $n$, (iv) $\tau$, (v) $\bar{\nu}_\mu$, (vi) $K^+$, (vii) $\Omega^-$, (viii) $\Xi^+$?

**d)** List all "messenger" particles in the Standard Model, i.e. those which carry (the known) fundamental forces.

**e)** The "elementary" particles made of the light quarks ($u$, $d$, $s$), or their antiquarks, can be organized in multiplets. Draw the diagrams for (i) the lightest pseudoscalar meson nonet, (ii) the lightest vector nonet, (iii) the baryon spin-$\frac{1}{2}$ octet, and (iv) the baryon spin-$\frac{3}{2}$ decuplet.

**f)** Write down the masses and lifetimes of all particles in the multiplets above. Which patterns do you find in these data? Where do you find the authorative source of such information on the internet?

**Problem 4. Processes in the Standard Model**

**a)** Which of the processes below do you believe are possible according to the Standard Model?

Which type of interaction is involved if the process is possible?

Which conservation law is involved if the process is impossible?

1. $p + \bar{p} \to \pi^+ + \pi^0$
2. $e^+ + e^- \to \mu^+ + \mu^-$
3. $e^+ \to \mu^+ + \nu_\mu + \bar{\nu}_e$
4. $\Sigma^- \to n + \pi^-$
5. $\mu^- \to e^- + \bar{\nu}_e$
6. $\bar{\nu}_e + p \to n + e^+$
7. $e + p \to \nu_e + \pi^0$
8. $\pi^0 \to \gamma + \gamma$
9. $\pi^0 \to \gamma + \gamma + \gamma$
10. $\pi^- \to \mu^- + \bar{\nu}_\mu$
11. $\pi^- \to e^- + \bar{\nu}_e$