FY3403 Particle physics
Problemset 6 fall 2008

NTNU
Institutt for
fysikk

Problem 10. The symmetry group of a square

In the lectures we studied the symmetry group of an equilatrial triangle. In this problem you should repeat this for a square.

a) How many elements are there in this group?

b) Find the multiplication table for this group.

c) Is this group abelian or non-abelian?

d) Find all subgroups of this group.

e) Can you find some factor groups (also called quotient groups) associated with this group?

Problem 11. Rotation of spin-$\frac{1}{2}$ states

The rotation operator $R(\alpha,\hat{n})$ for a spin-$\frac{1}{2}$ wave function (spinor) $\chi = \begin{pmatrix} a \\ b \end{pmatrix}$ is

$$R(\alpha,\hat{n}) = \mathrm{e}^{-\mathrm{i}\alpha\,\hat{n}\cdot\boldsymbol{\sigma}/2}\,, \tag{1}$$

where α is the rotation angle and the unit vector $\hat{n}$ is the rotation axis.

a) Explain geometrically why we should have $R(-\alpha,-\hat{n}) = R(\alpha,\hat{n})$.

b) Show, e.g. by series expansion of the exponential, that

$$R(\alpha,\hat{n}) = \cos(\alpha/2) - \mathrm{i}\sin(\alpha/2)\,\hat{n}\cdot\boldsymbol{\sigma}\ . \tag{2}$$

c) For a state with spin along the z-axis the corresponding spinor is $\chi = \chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Rotate this state an angle α about the y-axis, and show that the rotated spinor becomes $\chi_\alpha = \begin{pmatrix} \cos(\alpha/2) \\ \sin(\alpha/2) \end{pmatrix}$.

d) Show that χ_α is an eigenstate of $\sigma_{\hat{n}} \equiv \hat{n}\cdot\boldsymbol{\sigma}$, where $\hat{n}$ is the vector obtained by rotating the unit vector $\hat{e}_z$ an angle α about the y-axis. Draw a figure.