NTNU **Institutt for fysikk**

Midsemester exam in FY3403 PARTICLE PHYSICS

Wednesday October 21, 2009
12:15–14:00

Allowed help: Particle Physics Booklet, standard calculator

Remember to write your student number on every sheet.
Answers may be given in English or Norwegian (or Swedish, Danish, German or French).

This problem set consists of 3 pages.

Problem 1.

A particle of mass M decays into three particles of masses m_1, m_2, m_3. Let P, p_1, p_2, p_3 be the four-momenta of the decaying particle and of the three particles in the final state.
Conservation of energy and momentum means that $P = p_1 + p_2 + p_3$.
From these four-vectors we get Lorentz-invariant quantities such as

$$P^2 = (P^0)^2 - (P^1)^2 - (P^2)^2 - (P^3)^2 = M^2c^2 .$$

Define the invariant mass m_{12} by the formula $m_{12}^2\, c^2 = (p_1 + p_2)^2 = (P - p_3)^2$, and define m_{13} and m_{23} similarly.

Derive a relation between m_{12}^2 and the energy E_3 of particle 3 in the rest frame of the decaying particle.

Show that the sum $m_{12}^2 + m_{13}^2 + m_{23}^2$ is constant.

Problem 2.

The meson $\phi(1020)$ decays mainly into two K mesons, even though there is very little energy left over as kinetic energy in the final state. See the Meson Summary Table in the Particle Physics Booklet.
In the quark model the quark content of ϕ is $s\bar{s}$, and its decay mainly into K mesons is "explained" by the so called OZI rule (OZI for Okubo, Zweig and Iizuka).
The same rule is used to explain why the particle $J/\psi(3097)$, with quark content $c\bar{c}$, is exceptionally stable.

What is the OZI rule?

The next most frequent decay mode of ϕ is listed in the table as $\rho\pi + \pi^+\pi^-\pi^0$. The reason is clearly that it is difficult to distinguish between these final states.

Why is it difficult?

How would you try to distinguish between the final states $\rho\pi$ and $\pi^+\pi^-\pi^0$?

Problem 3.

Consider the following charge states of two π mesons: $|\,\pi^+\pi^-\rangle$, $|\,\pi^-\pi^+\rangle$ and $|\,\pi^0\pi^0\rangle$.
Use the table of Clebsch–Gordan coefficients, pages 273 and 274 in the Particle Physics Booklet, and write the decomposition of these states into states of different total isospin.

Problem 4.

The meson $\omega(782)$ has quantum numbers $I^G(J^{PC}) = 0^-(1^{--})$, see the Meson Summary Table.

$\omega(782)$ and $\phi(1020)$ are both called vector mesons. Why?

$\omega(782)$ has three decay modes with branching ratios higher than 1 %.
Which interaction is responsible for each of these three decay modes?
Explain your reasoning (even if you have to guess the answers).

The decay mode $\omega \to 3\pi^0$ is not observed, and an upper limit for the branching ratio is listed, under the heading "Charge conjugation (C) violating modes".
Explain why (how) it violates charge conjugation symmetry.

The decay mode $\omega \to 2\pi^0$ is also not observed. It violates charge conjugation symmetry just as much as $\omega \to 3\pi^0$, but is not listed under "C violating modes".
In fact, $\omega \to 2\pi^0$ is not listed at all in the table. The reason may be that it is forbidden by a fundamental conservation law (more fundamental than charge conjugation symmetry), together with the experimental result that π mesons are bosons. Explain!

You may want to reconsider this problem after answering the next problems.

Some notation used in the following:

One particle. Mass m, position $\vec{r}$, momentum $\vec{p}$, orbital angular momentum $\vec{L} = \vec{r} \times \vec{p}$.
A wave function $\psi = \psi(\vec{r})$ with definite orbital angular momentum satisfies the eigenvalue equations

$$\vec{L}^{\,2}\psi = \ell(\ell+1)\hbar^2\psi \;, \qquad L_z\,\psi = \mu\hbar\,\psi \;.$$

Possible values for the quantum numbers are $\ell = 0, 1, 2, \ldots$ and $\mu = -\ell, -\ell+1, \ldots, \ell$.
The angular momentum eigenfunction ψ has the symmetry property that $\psi(-\vec{r}) = (-1)^\ell\,\psi(\vec{r})$.

Two particles, numbered 1 and 2. Centre of mass position $\vec{R}$ and total momentum $\vec{P}$, relative position $\vec{r}$ and relative momentum $\vec{p}$, defined as

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} \;, \qquad \vec{P} = \vec{p}_1 + \vec{p}_2 \;, \qquad \vec{r} = \vec{r}_1 - \vec{r}_2 \;, \qquad \vec{p} = \frac{m_2\vec{p}_1 - m_1\vec{p}_2}{m_1 + m_2} \;.$$

Define $\vec{L}_{\mathrm{CM}} = \vec{R} \times \vec{P}$ and $\vec{L}_{\mathrm{rel}} = \vec{r} \times \vec{p}$, then the total orbital angular momentum is

$$\vec{L} = \vec{L}_1 + \vec{L}_2 = \vec{L}_{\mathrm{CM}} + \vec{L}_{\mathrm{rel}} \;.$$

We consider wave functions of the product form $\psi(\vec{r}_1, \vec{r}_2) = \psi_{\mathrm{CM}}(\vec{R})\,\psi_{\mathrm{rel}}(\vec{r})$. The centre of mass wave function ψ_{CM} is irrelevant here, we may simply take it to be constant, so that $\vec{L}_{\mathrm{CM}} = 0$ and $\vec{L} = \vec{L}_{\mathrm{rel}}$. The relative wave function ψ_{rel} has orbital angular momentum quantum number ℓ_{rel}.

Problem 5.

Let us consider the decay modes $\omega(782) \rightarrow 2\pi$, either $\pi^+\pi^-$ or $\pi^0\pi^0$.
We use always the rest frame of the decaying ω meson.
For the sake of the argument we choose to ignore the fact that $\pi^0\pi^0$ is forbidden.

What is the orbital angular momentum of the final state (ℓ_{rel} in the notation introduced above)?
We always assume that angular momentum is conserved.

It is possible to analyze the 2π final state without introducing the concept of isospin. Then we treat the π^+ and the π^- as distinguishable particles. The two π^0 mesons are identical bosons, and their wave function has to be symmetric, $\psi(\vec{r}_1, \vec{r}_2) = \psi(\vec{r}_2, \vec{r}_1)$, but for the sake of the argument we will ignore this symmetry requirement here.

What is the parity P of the final state?
Is the parity different for the final states $\pi^+\pi^-$ and $\pi^0\pi^0$?
Is P conserved in the decay?

What is the charge conjugation symmetry C of the final state?
Is the charge conjugation symmetry different for the final states $\pi^+\pi^-$ and $\pi^0\pi^0$?
Is C conserved in the decay?

Problem 6.

We consider still the decay modes $\omega(782) \rightarrow 2\pi$.
Now we introduce isospin, and we write the total wave function of the two π mesons as a product

$$\Psi(Q_1, \vec{r}_1; Q_2, \vec{r}_2) = \psi_{\text{isospin}}(Q_1, Q_2)\, \psi_{\text{space}}(\vec{r}_1, \vec{r}_2) \ .$$

Here $Q = I_3$ is the electric charge, I_3 being the third component of the isospin.
Then we treat the two π mesons as identical bosons, even if they have different charges, and we require the total wave function to be symmetric under interchange,

$$\Psi(Q_1, \vec{r}_1; Q_2, \vec{r}_2) = \Psi(Q_2, \vec{r}_2; Q_1, \vec{r}_1) \ .$$

The symmetry restriction on the total wave function restricts the total isospin in the final state.

What is the restriction on the total isospin?

Is the isospin conserved in the decay?