FY 8104 Symmetry in physics

Problem 1, 2007

1. **The symmetry group of an equilateral triangle**

   An equilateral triangle may be oriented in six different ways and still look the same.

   (a) Label the corners of the triangle, and list the six orientations.

   (b) Find the group C_{3v} of all transformations between these orientations.

   (c) Find the multiplication table for the transformation group C_{3v}.

   (d) A group G is *Abelian* if $gh = hg$ for all $g, h \in G$. Otherwise it is *non-Abelian.* Is C_{3v} Abelian or non-Abelian?

   (e) Find all the subgroups of C_{3v}, and decide whether thay are Abelian or non-Abelian.

   (f) Find for each of these subgroups (H_i) the corresponding decomposition of C_{3v} into left and right cosets:

   $$G = g_1 H_i \cup g_2 H_i \cup \cdots$$
   $$G = H_i g_1 \cup H_i g_2 \cup \cdots$$

   (g) Observe that for a given subgroup (H_i), every left or right coset has the same number of elements. Explain why.

   (h) Find the decomposition of C_{3v} into conjugation classes.

   (i) Why is the number of elements of a conjugation class in a finite group G always a divisor of the total number of elements (the *order* $|G|$) of the group?

   Given:

   (a) If G is a group, H a subgroup of G, and $g \in G$ then

   $$gH \equiv \{gh \mid h \in H\}$$

   is called a *left coset* of H, and

   $$Hg \equiv \{hg \mid h \in H\}$$

   is called a *right coset* of H. The Norwegian word for "coset" is "restklasse". Two left cosets $g_1 H$ and $g_2 H$ (or right cosets Hg_1 og Hg_2) are either identical or disjoint (containing no common elements).

   (b) If G is a group and $h \in G$ then

   $$C_h \equiv \left\{ghg^{-1} \mid g \in G\right\}$$

   is called the *conjugation class* of h in G. Two conjugation classes C_h and C_k are either identical or disjoint.