The Norwegian University of Science and Technology
Department of Physics

Contact person:
Name: Jan Myrheim
Telephone: 93653

Examination, course MNFFY250 Astrophysics
Tuesday June 3, 2003
Time: 09.00–15.00

Grades made public: Tuesday June 24, 2003

Allowed to use: Calculator, mathematical tables.

A table of physical constants is found at the end of this document.
All subproblems carry equal weights in grading.

Problem 1:

a) What is a Hertzsprung–Russell diagram (HR diagram)?
Sketch in a figure where in the HR diagram you will find the main sequence, giant stars, supergiants and white dwarf stars.
What about neutron stars?

b) What is common to all main sequence stars?
What distinguishes them, so that the main sequence becomes a narrow band in the HR diagram, and not just a point?

c) Where in the main sequence band in the HR diagram are those stars located that have the longest life times, and those that have the shortest life times?
Explain briefly how to use this in order to find the age of a star cluster.

Problem 2:

a) Describe briefly the properties of the three layers that the solar atmosphere consists of: the photosphere, the chromosphere, and the corona.

b) What are sun spots, and what is solar wind?
Inside sun spots there are strong magnetic fields.
How can one measure magnetic fields on the surface of the Sun?

c) The light from the photosphere has a continuous spectrum, with absorption lines, whereas the light from the chromosphere consists of emission lines.
What does this tell about the density and the temperature of the gas in the chromosphere, as compared to the photosphere?
Does it tell anything about temperature variations in the photosphere?

d) The energy radiated by the Sun is liberated in a series of nuclear reactons, called the proton–proton chain.
Which particles are the end products of the proton–proton chain, and which particles go into the reactions? There is no need to describe all the reactions in detail, just say which particles go in and which come out, when the reaction chain is seen as a whole.

e) Why does the proton–proton chain depend on the temperature being high?
The nuclear reactions take place only in the central regions of the Sun, because that is the only place where the temperature is sufficiently high. Why is the temperature highest in the centre?

f) Why are many elements, like gold, very rare compared to others, like oxygen?

Problem 3:

a) Kepler's third law states that

$$P^2 = \frac{4\pi^2 a^3}{G(m_1 + m_2)} ,$$

for two masses m_1 and m_2 bound by their mutual gravitational attraction, in such a way that the one moves in an elliptic orbit relative to the other one. a is the major half axis of the ellipse (the mean value of the largest and smallest distance between the two masses), whereas P is the orbital period, and G is the gravitational constant.
Derive Kepler's third law in the special case where one mass is much larger than the other one, say, $m_1 >> m_2$, and where the ellipse is a circle.

b) What is the smallest possible period for a planet in our solar system?
The Sun rotates once around its own axis in about 30 days.
What is the smallest possible rotation period for the Sun?

c) What is the smallest possible rotation period for the white dwarf star Sirius B, having the same mass as the Sun, and a radius of 5200 km?
The answer to this question proves that at least some pulsars can not be rotating white dvarfs. Why?

d) On August 28 this year Mars is closer to the Earth than it has been or will be for many years, the distance is then 56 million kilometer. We imagine a space ship being launched from the Earth half a year earlier (on February 28). We imagine the space ship following an elliptic orbit (not using its rocket engines) which is tangent to the orbits of both the Earth and Mars, so that perihelion (the smallest distance to the Sun) of its orbit is one astronomical unit, 150 million km, and aphelion (the largest distance to the Sun) is 206 million km.
How long time will this space ship use from the Earth orbit out to the Mars orbit?
If the space ship is launched on February 28, it will be late for its rendezvous with Mars. When will it have to be launched in order to meet Mars? Assume for simplicity that both the Earth and Mars move in circular orbits around the Sun, in the same plane.
At the moment when the space ship meets Mars, is its velocity (orbital velocity around the Sun) larger or smaller than the velocity of Mars?

Problem 4:

Let λ_0 be the wave length of a given spectral line when the light source is lying at rest in the laboratory where the wave length is measured. If the same spectral line from a star, or another astronomical light source, is observed with the wave length λ, its red shift is defined as

$$z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{\lambda}{\lambda_0} - 1 \ .$$

a) Why is the quantity z called red shift?
Negative red shift is also called blue shift. Why?

Red shift may be due to the Doppler effect: if the light source moves away with a velocity v, then

$$z = \frac{\sqrt{1 + \frac{v}{c}}}{\sqrt{1 - \frac{v}{c}}} - 1 \approx \frac{v}{c} \ .$$

Here c ic the speed of light in vacuum.
Red shift may also be due to the gravitational field: if the gravitational potential is ϕ_0 where the light source is located, and is ϕ_1 where the observer is located, then

$$z = \frac{\sqrt{1 + \frac{2\phi_1}{c^2}}}{\sqrt{1 + \frac{2\phi_0}{c^2}}} - 1 \approx \frac{\phi_1 - \phi_0}{c^2} \ .$$

The gravitational potential from a mass M at a distance r is

$$\phi = -\frac{GM}{r} \ ,$$

when the mass distribution is spherically symmetric, and the distance r is measured from its centre.

b) Compute the gravitational potential on the surface of the Earth.
You may have to take into account both the gravitational field of the Earth and the gravitational field of the Sun.
Which of the two contributions to the gravitational potential, from the Earth or from the Sun, is the largest? (Largest means here largest in absolute value.)

c) Compute the gravitational red shift of the spectral lines in the light from the white dvarf star Sirius B, having a mass equal to the solar mass and a radius of 5200 km.
Do you need to take into account that the gravitational potential on the surface of the Earth is nonzero?

d) When measuring the gravitational red shift in the light from Sirius B, one has to correct for the orbital velocity of the Earth in its orbit around the Sun. The Earth moves towards Sirius in September, and away from Sirius in March.

How much does the Doppler effect contribute to the red shift, in September and in March?
Assume for simplicity that Sirius does not move relative to the Sun.

e) In the points b), c) and d) above you may use the approximate formulae

$$z = \frac{v}{c}$$

for the Doppler effect, and

$$z = \frac{\phi_1 - \phi_0}{c^2}$$

for the gravitational red shift. Justify this claim.

f) Hubble's law states that $v = H_0 d$, where v is the velocity at which a galaxy moves away, d is the distance to the galaxy, and H_0 is a constant.
Assume that $H_0 = 70$ km/s per megaparsec (1 parsec is 3.26 light years). Assume also that the galaxies move away from each other at constant velocities, with negligible deceleration due to the gravitational attraction between them, and with negligible acceleration in the expansion of the universe due to a nonzero cosmological constant.
How long time ago is it that the universe was half its present size (in linear dimensions)? Compare with the age of the solar system.

g) The cosmological red shift is usually interpreted as a Doppler effect. But it may be interpreted alternatively in a third way: if the wave length has increased by a factor $\lambda/\lambda_0 = 1 + z$, this is because the universe has expanded by the same factor $1 + z$.
The cosmic background radiation at present has a temperature of 2.73 K.
What temperature did it have when the universe was half its present size?
(Use e.g. Wien's law, that the wave length at which the intensity of black body radiation is maximal, is inversely proportional to the temperature.)

Some useful constants

Newton's gravitational constant:	$G = 6.673 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
The speed of light in vacuum:	$c = 299\,792\,458\,\mathrm{m/s}$
The permeability of vacuum:	$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2}$
The permittivity of vacuum:	$\epsilon_0 = 1/(\mu_0 c^2) = 8.854 \times 10^{-12}\,\mathrm{F/m}$
The reduced Planck's constant:	$\hbar = h/(2\pi) = 1.055 \times 10^{-34}\,\mathrm{J\,s}$
The elementary charge:	$e = 1.602 \times 10^{-19}\,\mathrm{C}$
The fine structure constant:	$\alpha = e^2/(4\pi\epsilon_0\hbar c) = 1/137.036$
The electron mass:	$m_\mathrm{e} = 9.109 \times 10^{-31}\,\mathrm{kg} = 0.511\,\mathrm{MeV}/c^2$
The proton mass:	$m_\mathrm{p} = 1.6726 \times 10^{-27}\,\mathrm{kg} = 938.28\,\mathrm{MeV}/c^2$
The neutron mass:	$m_\mathrm{n} = 1.6749 \times 10^{-27}\,\mathrm{kg} = 939.57\,\mathrm{MeV}/c^2$
The mass of the Earth:	$M_\oplus = 6.0 \times 10^{24}\,\mathrm{kg}$
The radius of the Earth:	$R_\oplus = 6.4 \times 10^3\,\mathrm{km}$
The mass of the Sun:	$M_\odot = 2.0 \times 10^{30}\,\mathrm{kg}$
The radius of the Sun:	$R_\odot = 7.0 \times 10^5\,\mathrm{km}$
The distanc to the Sun (one astronomical unit):	$1\,\mathrm{AU} = 1.50 \times 10^8\,\mathrm{km}$